1 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

$A^2 = (N+1)(N+1)$
$= N^2 + 2N + 1$ (1)

$B^2 = (N-1)(N-1)$
$= N^2 - 2N + 1$

$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$
$= N^2 - N^2 + 2N + 2N + 1 - 1$
$= 4N$ (1)

since N is a multiple of 5,
$4N$ is a multiple of 20. (1)

$\therefore A^2 - B^2$ is always a multiple of 20.

(Total for Question 1 is 3 marks)

2 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

$(n+1)^2 - (n)^2$ 1

$= n^2 + 2n + 1 - n^2$ 1

$= 2n + 1$

1

$\therefore$ $2n+1$ will always be odd for any n values.

(Total for Question 2 is 3 marks)

3 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

Let 3 consecutive even numbers : $2n$, $2n+2$, $2n+4$ (1)

Difference between square of largest and smallest number :

$(2n+4)^2 - (2n)^2$ (1)

$= (4n^2 + 16n + 16) - 4n^2$

$= 16n + 16$

$= 8(2n+2)$ (1)

$\therefore$ 8 times the middle number (shown)

(Total for Question 3 is 3 marks)

4 Prove algebraically that the product of any two odd numbers is always an odd number.

Let odd number 1 be = $2n+1$

Let odd number 2 be = $2m+1$

$(2n+1)(2m+1) = 4mn + 2n + 2m + 1$ ①

② $= 2(2mn + n + m) + 1$

this will make sure the result will always be odd.

①

Since the product of 2 odd numbers is $2(2mn+n+m)+1$, this proves that the result will always be an odd number.

(Total for Question 4 is 4 marks)

5 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

let n, $n+1$, $n+2$ 1

$n^2 = n^2$

$(n+1)^2 = n^2 + 2n + 1$

$(n+2)^2 = n^2 + 4n + 4$

1

$n^2 + n^2 + 4n + 4 = 2n^2 + 4n + 4$

$2(n^2 + 2n + 1) + 2 = 2n^2 + 4n + 2 + 2$

$= 2n^2 + 4n + 4$ (proved)

1

(Total for Question 5 is 3 marks)

6 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

let $2n+1$ and $2n-1$

$(2n+1)^2 = 4n^2+4n+1$ (1)
$(2n-1)^2 = 4n^2-4n+1$

$(4n^2+4n+1)+(4n^2-4n+1) = 8n^2+2$

(1)

$8n^2+\underline{2}$ (1)

(Total for Question 6 is 3 marks)

7 Prove algebraically that, for any three consecutive even numbers,

the sum of the squares of the smallest even number and the largest even number is 8 more than twice the square of the middle even number.

let $2n$, $2n+2$, $2n+4$ (1)

$(2n)^2 = 4n^2$

$(2n+2)^2 = 4n^2 + 8n + 4$ (1)

$(2n+4)^2 = 4n^2 + 16n + 16$

(1) $(2n)^2 + (2n+4)^2 = 4n^2 + 4n^2 + 16n + 16$

$= 8n^2 + 16n + 16$

(2) $2 \times (2n+2)^2 = 2(4n^2 + 8n + 4)$

$= 8n^2 + 16n + 8$

(1)

(1) − (2) : $8n^2 + 16n + 16 - 8n^2 + 16n + 8$

$= 8$ (proven)

(Total for Question 7 is 3 marks)